IP/BBSR/96-45

hep-th/9609015

On Orientifold Constructions of Type IIA Dual Pairs

Anindya K. Biswas*
Institute of Physics,
Bhubaneswar 751 005, INDIA

February 1, 2008

Abstract

In this paper we analyze the earlier constructions of the type IIA dual pairs through orientifolding. By an appropriate choice of Γ-matrix basis for the spinor representations of the U-duality group, we give an explicit relationship between the orientifold models and their dual pairs.

The constructions of "strong-weak" dual pair [1] of string theories, and in particular the construction of type II dual pair of string theories in various dimensions have been investigated by several authors,[2, 3, 4]. In these constructions, the $SO(5,5;Z)$ U-duality symmetry of the six dimensional type II string theory was used to construct several dual pair of string theories upon further compactifications of the extra dimensions. The matching of the massless spectra in four dimensions was observed as an evidence that these theories are indeed dual to each other. Other type II dual pair models include the orientifold examples in four dimensions[4] and two dimensional examples[5] . Recently a U-duality invariant partition function for these models have been proposed which gives the degeneracy of the fundamental as well as the solitonic states[6].

In an earlier paper, a class of type II dual pair models were constructed through orientifolding[4]. It was found that the type IIA string theory in ten dimensions possesses certain discrete symmetries which cannot be embedded into the T-duality group in lower dimensions. However, it was argued that two such discrete symmetries

*akb@iopb.ernet.in

are related by "strong-weak" duality and can be used for model building. In this paper we examine the relationship among these discrete symmetries in detail. This is achieved by arranging the fields into appropriate representations of the U-duality group viz. $SO(5,5;Z)$ and finding their transformations. Interestingly, we find that in the present case the U-duality relates the discrete symmetries which are themsleves not the elements of the $SO(5,5)$ symmetry.

Let us now start by briefly reviewing the discrete symmetries of the type IIA theory as discussed in [4]. The bosonic field content of the type IIA theory in ten dimensions are the graviton G_{MN}, antisymmetric tensor B_{MN}, dilaton ϕ, the one-form field A_M and the three-form field C_{MNP}. Among these, the gaviton, dilaton and the antisymetric tensor fields arise from the NS-NS sector of the theory and the one and three-form fields from the R-R sector. This theory has several discrete symmetries. The ones known as "T-duality" have been studied extensively. One can, however, show that there are other discrete symmetries present as well. One such symmetry referred to as Z_2^0 is used for constructing orientifold models. Under this, string worldsheet changes its orientation. At the same time, there is a change of sign for the coordinates, $(X^5, ..., X^9)$. Another symmetry, relevant for us in this paper is an improper T- duality rotation. This will be referred to as $Z_2^\star$ and acts as $(X^5, .., X^8) \rightarrow -(X^5, .., X^8)$. Both Z_2^0 and $Z_2^\star$ remain the symmetries of the type IIA action upon toroidal compactifications. In particular, for the compactifications to four dimensions and on the basis of strong analogy with the results of [3], it was pointed out in [4] that by modding out the original theory by these discrete symmetries leads to models which are "strong-weak" dual pair.

We now follow the general approach of references [3, 4] and show that the two discrete symmetries discussed in the last paragraph are connected by a U-duality symmetry. The particular U-duality element which connects these is, however, different than that in [4] by a T-duality factor. This is still, however, an element of $SO(5,5)$ and has a ten dimensional matrix representation. We now obtain this representation by arranging the ten dimensional fields in terms of U-duality multiplets $Q_{\bar\mu\bar\nu\bar\rho}$, $\tilde{M}$ and $P_{\bar\mu}$ as:

$$Q_{\bar\mu\bar\nu\bar\rho} \equiv \begin{pmatrix} H_{\bar\mu\bar\nu\bar\rho} \\ e^{-2\phi}\tilde{H}_{\bar\mu\bar\nu\bar\rho} \\ F_{\bar\mu\bar\nu\bar\rho 6} \\ F_{\bar\mu\bar\nu\bar\rho 7} \\ F_{\bar\mu\bar\nu\bar\rho 8} \\ F_{\bar\mu\bar\nu\bar\rho 9} \\ F'_{\bar\mu\bar\nu\bar\rho 6} \\ F'_{\bar\mu\bar\nu\bar\rho 7} \\ F'_{\bar\mu\bar\nu\bar\rho 8} \\ F'_{\bar\mu\bar\nu\bar\rho 9} \end{pmatrix} \equiv \begin{pmatrix} H_{\bar\mu\bar\nu\bar\rho} \\ e^{-2\phi}\tilde{H}_{\bar\mu\bar\nu\bar\rho} \\ \vec{D}_{\bar\mu\bar\nu\bar\rho} \end{pmatrix},$$

$$\tilde{M} = \begin{pmatrix} e^{2\phi} & -1/2e^{2\phi}\psi^T L\psi & -e^{2\phi}\psi^T \\ & \begin{matrix} e^{-2\phi}+\psi^T L R_s(M) L\psi \\ +1/4e^{2\phi}(\psi^T L\psi)^2 \end{matrix} & \begin{matrix} \psi^T L R_s(M) \\ +1/2e^{2\phi}\psi^T(\psi^T L\psi) \end{matrix} \\ & & R_s(M)+e^{2\phi}\psi\psi^T \end{pmatrix}, P_{\bar{\mu}} = \begin{pmatrix} C_{\bar{\mu}69} \\ C_{\bar{\mu}79} \\ C_{\bar{\mu}89} \\ K'_{\bar{\mu}} \\ C_{\bar{\mu}78} \\ C_{\bar{\mu}68} \\ C_{\bar{\mu}67} \\ A_{\bar{\mu}} \\ g_{\bar{\mu}6} \\ g_{\bar{\mu}7} \\ g_{\bar{\mu}8} \\ g_{\bar{\mu}9} \\ B_{\bar{\mu}6} \\ B_{\bar{\mu}7} \\ B_{\bar{\mu}8} \\ B_{\bar{\mu}9} \end{pmatrix}, \quad (1)$$

where F and F' in the expression for Q are the original and dual field strengths of the six dimensional RR 3-form field C. These field strengths are related by $\tilde{Q} = \tilde{M}\tilde{L}\,Q + O(\psi)$ [3]. Hence to zeroeth order in ψ, H and $\tilde{H}$ are Poincare dual. In eq.(1) only upper triangular entries of $\tilde{M}$ has been written, as this is a symmetric matrix. $\tilde{M}$ contains M and ψ. M is defined in terms of toroidal moduli g, B and ψ in terms of various RR scalars. They are given as,

$$M = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g-Bg^{-1}B \end{pmatrix}, \quad g = \begin{pmatrix} g_{mn} & g_{m9} \\ g_{9n} & g_{99} \end{pmatrix}, \quad B = \begin{pmatrix} B_{mn} & B_{m9} \\ B_{9n} & 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} A_6 \\ . \\ . \\ A_9 \\ C_{789} \\ C_{689} \\ C_{769} \\ C_{678} \end{pmatrix},$$

where; m,n = 6,., 8. $\tilde{M}$ is an $SO(5,5)$ matrix i.e. it satisfies the relation, $\tilde{M}\tilde{L}\tilde{M} = \tilde{L}$ where,

$$\tilde{L} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \tilde{L}_8 \end{pmatrix}, \quad \tilde{L}_8 = \begin{pmatrix} 0 & I_4 \\ I_4 & 0 \end{pmatrix}. \quad (2)$$

In addition, we have g_{55} and the graviton $g_{\mu\nu}$.

Then symmetries defined as Z_2^0 and $Z_2^\star$ do not transform the fields $g_{\mu\nu}$, g_{55} as well as dilaton ϕ. For describing other transformations, in the following, we will use barred(unbarred) indices for six(five) dimensional coordinates. As our objective will be to relate the two discrete symmetries we consider Z_2^0 and $Z_2^\star$ operations on two separate multiplets rather than on the same multiplet. We will denote the two multiplets

as primed and unprimed respectively. We will add superscripts 0 and $\star$ respectively on them to denote transformed multiplets. Z_2^0 transforms unprimed multiplets as follows:

$$Q^0_{\bar\mu\bar\nu\bar\rho} = \begin{pmatrix} -H_{\mu\nu\rho}, & H_{\mu\nu5} \\ e^{-2\phi}\tilde{H}_{\mu\nu\rho}, & -e^{-2\phi}\tilde{H}_{\mu\nu5} \\ F_{\mu\nu\rho6}, & -F_{\mu\nu56} \\ F_{\mu\nu\rho7}, & -F_{\mu\nu57} \\ F_{\mu\nu\rho8}, & -F_{\mu\nu58} \\ F_{\mu\nu\rho9}, & -F_{\mu\nu59} \\ -F'_{\mu\nu\rho6}, & F'_{\mu\nu56} \\ -F'_{\mu\nu\rho7}, & F'_{\mu\nu57} \\ -F'_{\mu\nu\rho8}, & F'_{\mu\nu58} \\ -F'_{\mu\nu\rho9}, & F'_{\mu\nu59} \end{pmatrix}, \psi^0 = \begin{pmatrix} -A_6 \\ -. \\ -. \\ -A_9 \\ C_{789} \\ C_{689} \\ C_{769} \\ C_{678} \end{pmatrix}, P^0_{\bar\mu} = \begin{pmatrix} -C_{\mu69}, & C_{569} \\ -C_{\mu79}, & C_{579} \\ -C_{\mu89}, & C_{589} \\ K'_\mu, & -K'_5 \\ -C_{\mu78}, & C_{578} \\ -C_{\mu68}, & C_{568} \\ -C_{\mu67}, & C_{567} \\ A_\mu, & -A_5 \\ -g_{\mu6}, & g_{56} \\ -g_{\mu7}, & g_{57} \\ -g_{\mu8}, & g_{58} \\ -g_{\mu9}, & g_{59} \\ B_{\mu6}, & -B_{56} \\ B_{\mu7}, & -B_{57} \\ B_{\mu8}, & -B_{58} \\ B_{\mu9}, & -B_{59} \end{pmatrix}.$$

$$g^0 = g, \quad B^0 = -B. \tag{3}$$

We can rewrite the above transformations in matrix form as in Ref.[3]:

$$Q^0_{\bar\mu\bar\nu\bar\rho} \equiv \Omega(Z_2^0) Q_{\bar\mu\bar\nu\bar\rho}, \quad \tilde{M}^0 \equiv \Omega(Z_2^0)\tilde{M}\Omega(Z_2^0)^T, \quad P^0_{\bar\mu} \equiv \tilde{R}_s(Z_2^0) P_{\bar\mu},$$

where,

$$\Omega(Z_2^0) = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & I_4 & \\ & & & -I_4 \end{pmatrix}, \quad \tilde{R}_s(Z_2^0) = \begin{pmatrix} -I_3 & & & & & \\ & 1 & & & & \\ & & -I_3 & & & \\ & & & 1 & & \\ & & & & -I_4 & \\ & & & & & I_4 \end{pmatrix}. \tag{4}$$

provided

$$R_s(\Omega^0) = \Omega^0 \tag{5}$$

where, $\Omega^0 = \begin{pmatrix} I_4 & \\ & -I_4 \end{pmatrix}$.

Similarly the action of $Z_2^\star$ on primed multiplets are as follows:

$$(Q'_{\bar\mu\bar\nu\bar\rho})^\star = \begin{pmatrix} H_{\mu\nu\rho}, & -H_{\mu\nu5} \\ -e^{-2\phi}\tilde H_{\mu\nu\rho}, & e^{-2\phi}\tilde H_{\mu\nu5} \\ -F_{\mu\nu\rho6}, & F_{\mu\nu56} \\ -F_{\mu\nu\rho7}, & F_{\mu\nu57} \\ -F_{\mu\nu\rho8}, & F_{\mu\nu58} \\ F_{\mu\nu\rho9}, & -F_{\mu\nu59} \\ F'_{\mu\nu\rho6}, & -F'_{\mu\nu56} \\ F'_{\mu\nu\rho7}, & -F'_{\mu\nu57} \\ F'_{\mu\nu\rho8}, & -F'_{\mu\nu58} \\ -F'_{\mu\nu\rho9}, & F'_{\mu\nu59} \end{pmatrix}', (\psi')^\star = \begin{pmatrix} -A_6 \\ -. \\ -. \\ A_9 \\ C_{789} \\ C_{689} \\ C_{769} \\ -C_{678} \end{pmatrix}', (P'_{\bar\mu})^\star = \begin{pmatrix} -C_{\mu69}, & C_{569} \\ -C_{\mu79}, & C_{579} \\ -C_{\mu89}, & C_{589} \\ -K'_\mu, & K'_5 \\ C_{\mu78}, & -C_{578} \\ C_{\mu68}, & -C_{568} \\ C_{\mu67}, & -C_{567} \\ A_\mu, & -A_5 \\ -g_{\mu6}, & g_{56} \\ -g_{\mu7}, & g_{57} \\ -g_{\mu8}, & g_{58} \\ g_{\mu9}, & -g_{59} \\ -B_{\mu6}, & B_{56} \\ -B_{\mu7}, & B_{57} \\ -B_{\mu8}, & B_{58} \\ B_{\mu9}, & -B_{59} \end{pmatrix}'.$$

$$g^\star = \begin{pmatrix} g_{mn} & -g_{m9} \\ -g_{9n} & g_{99} \end{pmatrix}, \quad B^\star = \begin{pmatrix} B_{mn} & -B_{m9} \\ -B_{9n} & 0 \end{pmatrix}. \tag{6}$$

Again we can rewrite the above transformations in the matrix form

$(Q'_{\bar\mu\bar\nu\bar\rho})^\star \equiv \Omega(Z_2^\star)\, Q'_{\bar\mu\bar\nu\bar\rho}, \quad (\tilde M')^\star \equiv \Omega(Z_2^\star)\, \tilde M'\, \Omega(Z_2^\star)^T, \quad (P'_{\bar\mu})^\star \equiv \tilde R_s(Z_2^\star)\, P'_{\bar\mu}$

where

$$\Omega(Z_2^\star) = \begin{pmatrix} 1 & & & & & \\ & -1 & 0 & 0 & 0 & 0 \\ & 0 & -I_3 & 0 & 0 & 0 \\ & 0 & 0 & 1 & 0 & 0 \\ & 0 & 0 & 0 & I_3 & 0 \\ & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \tilde R_s(Z_2^\star) = \begin{pmatrix} -I_4 & & & & & \\ & I_4 & & & & \\ & & -I_3 & & & \\ & & & 1 & & \\ & & & & -I_3 & \\ & & & & & 1 \end{pmatrix} \tag{7}$$

provided once again

$$R_s(\Omega^\star) = \Omega^\star \tag{8}$$

with $\Omega^\star = \begin{pmatrix} -I_3 & & & \\ & 1 & & \\ & & I_3 & \\ & & & -1 \end{pmatrix}$.

Now let us verify eqns (3) and (5). We note that Ω^0, $\Omega^\star$ and corresponding $R_s(\Omega)$'s, act on the multiplet basises of Ref.[4]. They are $SO(8)$ matrices rather than $SO(4,4)$ ones. We apply $(\eta')^T$ on them to transform to matrices acting on the multiplet basises of Ref.[3], where $\eta' = (1/\sqrt{2})\begin{pmatrix} I_4 & I_4 \\ -I_4 & I_4 \end{pmatrix}$. Formally, we write the

transformed matrices acting on the multiplet basises of Ref.[3] as Ω and R_s. Then we embed them in $SO(10)$ representation as:

$$\bar{R}_s(\bar{\Omega}) = \begin{pmatrix} R_c(\Omega) & \\ & \Omega \end{pmatrix}, \quad \bar{\Omega} = \begin{pmatrix} I_2 & \\ & R_s \end{pmatrix}. \tag{9}$$

Bars denote either an $SO(10)$ or an $SO(5,5)$ representation acting on the multiplet bases of Ref.[3] depending on the metric involved. Then we see that the equation

$$(\bar{R}^{32}_s)(\bar{\Gamma}^s_c)^m(\bar{R}^{32}_s)^{-1} = (\bar{\Omega}^{-1})^m_n(\bar{\Gamma}^s_c)^n \tag{10}$$

is satisfied consistently when we use eqns. (5) and (8) and $\bar{R}^{32}_s = I_2 \otimes \bar{R}_s(\Omega)$. The relation (10) is a generalisation of our familiar relation $S^{-1}\gamma^\mu S = g^{\mu\nu}\gamma_\nu$ in 4-d quantum mechanics[7]. As an aside we note that the relation (10) gives back,

$$R_c(\Omega^0) = \begin{pmatrix} & & I_2 & 0 \\ & & 0 & -\sigma_3 \\ I_2 & 0 & & \\ 0 & -\sigma_3 & & \end{pmatrix}, \quad R_c(\Omega^\star) = \begin{pmatrix} & & I_2 & 0 \\ & & 0 & -I_2 \\ I_2 & 0 & & \\ 0 & -I_2 & & \end{pmatrix} \tag{11}$$

In the relation (10)

$$(\bar{\Gamma}^s_c)^m = \bar{\eta}^m_n\,(\Gamma^s_c)^n, \qquad (\Gamma^s_c)^m = (I_2 \otimes U^s_{16})\Gamma^m_W(I_2 \otimes U^s_{16})^T, \tag{12}$$

with

$$\bar{\eta} = (1/\sqrt{2})\begin{pmatrix} 1 & -1 & \\ 1 & 1 & \\ & & I_8 \end{pmatrix}.$$

They satisfy Clifford algebra:

$$\{(\bar{\Gamma}^s_c)^m, (\bar{\Gamma}^s_c)^n\} = 2\bar{L}^{mn}_c, \quad \{(\Gamma^s_c)^m, (\Gamma^s_c)^n\} = 2I^{mn}_{10}, \quad \{\Gamma^m_W, \Gamma^n_W\} = 2I^{mn}_{10}, \tag{13}$$

with $\bar{L}_c = \begin{pmatrix} \sigma_1 & \\ & I_8 \end{pmatrix}$. U^s_{16} in eq.(12) is given by

$$U^s_{16} = \begin{pmatrix} A & -A.\epsilon \otimes \epsilon \otimes \epsilon \\ A.\sigma \otimes I_2 \otimes \epsilon & A.\sigma_3 \otimes \epsilon \otimes I_2 \end{pmatrix}, A = \begin{pmatrix} 1 & 0 & 0 & -1 & 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}, \tag{14}$$

Γ_W's in eq.(12) are gamma matrices in a Weyl basis and are given explicitly as,

$$
\begin{aligned}
(\Gamma_W)^0 &= \sigma_2 \otimes I_{16}, & (\gamma_8)^1 &= \epsilon \otimes \epsilon \otimes \epsilon, \\
(\Gamma_W)^{1-5} &= \sigma_1 \otimes \epsilon \otimes (\gamma_8)^{1-5}, & (\gamma_8)^2 &= 1 \otimes \sigma_1 \otimes \epsilon, \\
(\Gamma_W)^{6-7} &= \sigma_1 \otimes \epsilon \otimes (\gamma_8)^{6-7}, & (\gamma_8)^3 &= 1 \otimes \sigma_3 \otimes \epsilon, \\
(\Gamma_W)^8 &= -\sigma_1 \otimes \sigma_1 \otimes I_8, & (\gamma_8)^4 &= \sigma_1 \otimes \epsilon \otimes 1, \\
(\Gamma_W)^9 &= \sigma_1 \otimes \sigma_3 \otimes I_8, & (\gamma_8)^5 &= \sigma_3 \otimes \epsilon \otimes 1, \\
(\Gamma_W)^{11} &= -\sigma_3 \otimes I_{16}, & (\gamma_8)^6 &= \epsilon \otimes 1 \otimes \sigma_1, \\
& & (\gamma_8)^7 &= \epsilon \otimes 1 \otimes \sigma_3
\end{aligned}
$$

with $\epsilon = i\sigma_2$. A set of $SO(1,9)$ gamma matrix representations in Dirac basis could be found in Ref.[8, 9].

We note that once we replace $(\bar{\Gamma}^s_c, \Gamma^s_c, \bar{L}_c)$ by the corresponding noncompact ones $(\bar{\Gamma}^s, \Gamma^s, \bar{L})$ in eqns. (10, 12, 13), eq.(10) holds true, if we substitute $\bar{\Omega}$ and $\bar{R}^s(\bar{\Omega})$ of eq. (9) by any T-duality element embedded in $SO(5,5)$ or the U-duality element $\bar{\Omega}_0$, of Ref.[3]. Γ^s's are related to Γ^s_c's as: $(\Gamma^s)^j = i(\Gamma^s_c)^j$ for j=1,...,5; otherwise,they are same. Here, $\bar{L} = \begin{pmatrix} \sigma_1 & \\ & \bar{L}_8 \end{pmatrix}$, $\bar{L}_8 = \begin{pmatrix} -I_4 & \\ & I_4 \end{pmatrix}$.

Now let us come back to eqns. (5) and (8). We find that,

$$
\begin{aligned}
\Omega(Z_2^0)\, \tilde{L}\, \Omega(Z_2^0)^T = -\tilde{L}, &\quad det\, \Omega(Z_2^0) = -1 \\
\Omega(Z_2^\star)\, \tilde{L}\, \Omega(Z_2^\star)^T = -\tilde{L}, &\quad det\, \Omega(Z_2^\star) = -1.
\end{aligned} \tag{15}
$$

Eq.(15) implies that $\Omega(Z_2^0)$ and $\Omega(Z_2^\star)$ do not correspond to $SO(5,5)$ transformations. To investigate the relation between the two symmetries, we note that Z_2^0 acting on $(Q, P, \tilde{M})$ and $Z_2^\star$ acting on $(Q^{'}, P^{'}, \tilde{M}^{'})$ separately are symmetries of action where Q and Q$^{'}$ etc. act as dummy symbols for multiplets of field variables appearing in the action. Now to establish their relationship let us assume that the action of Z_2^0 on $(Q, P, \tilde{M})$ and that of $Z_2^\star$ on $(Q^{'}, P^{'}, \tilde{M}^{'})$ are related by a symmetry element belonging to the equation of motion and see that this assumption is consistent or, in other words, we do not face any contradiction. Now we see that this assumption enables us to write:

$$
\begin{aligned}
Q^{'} = \tilde{\Omega} Q, \quad P^{'} = \tilde{R}_s\, P, \quad \tilde{M}^{'} = \tilde{\Omega}\, \tilde{M}\, \tilde{\Omega}^T, \\
\Omega(Z_2^\star) = \tilde{\Omega}\, \Omega(Z_2^0)\, \tilde{\Omega}^{-1}, \quad \tilde{R}_s(Z_2^\star) = \tilde{R}_s \tilde{R}_s(Z_2^0) \tilde{R}_s^{\ -1},
\end{aligned} \tag{16}
$$

where, $\tilde{\Omega}$ and $\tilde{R}_s$ are relating matrix transformations of Z_2^0 and $Z_2^\star$ on vector and spinor multiplets respectively. We find the explicit form of $\tilde{\Omega}$, $\tilde{R}_s$ as:

$$
\tilde{\Omega} = \begin{pmatrix} \sigma_1 & & & & \\ & 0 & 0 & I_3 & 0 \\ & 0 & 1 & 0 & 0 \\ & I_3 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{17}
$$

$$\tilde{R}_s = -\begin{pmatrix} 0 & I_8 \\ I_8 & 0 \end{pmatrix}, \tag{18}$$

where $\tilde{\Omega}$ satisfies $\tilde{\Omega}\tilde{L}\tilde{\Omega}^T = \tilde{L}$ and $det\tilde{\Omega} = 1$. This implies that $\tilde{\Omega}$ in eq.(17) is an $SO(5,5)$ vector transformation. It further implies that $Q, \tilde{M}$ and $Q^{'}, \tilde{M}^{'}$ satisfy the same equation of motion. So here, we do not face any contradiction. Then our task is to show that $\tilde{R}_s$ also belongs to U-duality symmetry $SO(5,5)$ and it corresponds to the spinor representation of the element whose vector representation is $\tilde{\Omega}$.

Now let us start by making some observation about the spinor multiplets $P_{\bar{\mu}}$ and $P^{'}_{\bar{\mu}}$. They are in 16-dimensional forms making it essential for us to consider our $SO(5,5)$ spinor representation in a Weyl basis where all generators in the 32-dimensional spinor representation reduce [8, 10] to two 16-dimensional blocks along the diagonal. The entries of the two blocks are either same or differ by an overall minus sign. Now we can find any Weyl basis for gamma matrix representation which keeps the form of $P_{\bar{\mu}}$ unchanged and satisfy the relation

$$(\tilde{R}_s^{\,32})\tilde{\Gamma}^m(\tilde{R}_s^{\,32})^{-1} = (\tilde{\Omega}^{-1})^m_n\tilde{\Gamma}^n. \tag{19}$$

where in (19), $\tilde{R}_s^{\,32} = I_2 \otimes \tilde{R}_s$ i.e. $\tilde{R}_s^{32} = \tilde{R}_s(\tilde{\Omega})$, thereby proving that $\tilde{R}_s$ in eq.(18) corresponds to the spinor representation of the element whose vector representation is $\tilde{\Omega}$.

Now instead of choosing an arbitrary gamma matrix representation in this way let us go about in a bit roundabout path to determine one representation which will be useful for us when we will be considering simultaneous modding out by 'Orientifold' pair of discrete symmetries and T-duality symmetries of Ref.[3]. Let us start by noting that $\tilde{\Omega} = \tilde{\Omega}_0\,\tilde{L}^{10}$, (throughout the paper tilde refers to objects taken in metric $\tilde{L}$ of Ref.[3]) with

$$\tilde{\Omega}_0 = \begin{pmatrix} \sigma_1 & & & & \\ & I_3 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 1 \\ & 0 & 0 & I_3 & 0 \\ & 0 & 1 & 0 & 0 \end{pmatrix}, \; \tilde{L}^{10} = \begin{pmatrix} I_2 & \\ & \tilde{L}_8 \end{pmatrix}. \tag{20}$$

In eqn.(20), $\tilde{\Omega}_0$ is obtained from $\bar{\Omega}_0$ in ref.[3] as:

$$\tilde{\Omega}_0 = \eta^{10}\,\bar{\Omega}_0\,(\eta^{10})^T, \quad \eta^{10} = \begin{pmatrix} I_2 & \\ & \eta^{'} \end{pmatrix}, \quad \eta^{'} = 1/\sqrt{2}\begin{pmatrix} I_4 & I_4 \\ -I_4 & I_4 \end{pmatrix}, \tag{21}$$

where η^{10} takes into account the metric change from ref.[3] to ref.[4]. Here $\tilde{L}^{10}$ is a T-duality element. It is obtained from $\bar{L}^{10}$ in the same way as $\tilde{\Omega}_0$ from $\bar{\Omega}_0$. Now we obtain $\tilde{R}_s$ in eq.(18) from $\bar{R}_s(\bar{\Omega}_0\,\bar{L}^{10})$ as:

$$\tilde{R}_s = (V_{16}\,\eta^{16})\,\bar{R}_s(\bar{\Omega}_0\,\bar{L}^{10})\,(V_{16}\,\eta^{16})^T, \tag{22}$$

where,

$$\bar{R}_s(\Omega_0) = \begin{pmatrix} 0 & I_8 \\ -I_8 & 0 \end{pmatrix}, \quad \bar{R}_s(\bar{L}^{10}) = \begin{pmatrix} \bar{L}_8 & \\ & -\bar{L}_8 \end{pmatrix}.$$

So the spinor multiplet in Ref.[4] is related to that in Ref.[3] as: $P_{\bar{\mu}} = (V_{16}\,\eta^{16})\,\bar{P}_{\bar{\mu}}$. As a result $\tilde{R}_s$ relates two spinor multiplets which satisfy the same equation of motion as $\bar{R}_s(\bar{\Omega}_0)$ and $\bar{R}_s(\bar{L}^{10})$ do. Infact, we verify that $\tilde{R}_s$ and $\tilde{\Omega}$ refer to same $SO(5,5)$ element as they satisfy

$$(\tilde{R}_s^{32})\tilde{\Gamma}^m(\tilde{R}_s^{32})^{-1} = (\tilde{\Omega}^{-1})^m_n\tilde{\Gamma}^n. \tag{23}$$

This is consistent with our initial assumption. In eq.(23), $\tilde{R}_s^{32} = I_2 \otimes \tilde{R}_s$ and

$$\tilde{\Gamma}^m = \tilde{\eta}^m_n\Gamma^n, \quad \Gamma^m = (V\eta^{32})(\Gamma^s)^m(V\eta^{32})^T, \quad \eta^{32} = I_2 \otimes \eta^{16}, \quad V = I_2 \otimes V_{16},$$

$$\tilde{\eta} = (1/\sqrt{2})\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & I_4 & I_4 \\ 0 & 0 & -I_4 & I_4 \end{pmatrix}, \quad \eta^{16} = \begin{pmatrix} \eta' & \\ & \eta' \end{pmatrix}, \quad V_{16} = \begin{pmatrix} I_4 & & & \\ & I_4 & & \\ & & 0 & I_4 \\ & & I_4 & 0 \end{pmatrix}. \tag{24}$$

Γ's satisfy the Clifford algebra:

$$\{\tilde{\Gamma}^m, \tilde{\Gamma}^n\} = 2\tilde{L}^{mn}, \quad \{\Gamma^m_W, \Gamma^n_W\} = 2l^{mn}, \tag{25}$$

where, $l^{mn} = diag(1, -I_5, I_4)$. It is interesting to note here that we do not face any contradiction to the assumption that the two discrete symmetries are related and we find that they are related by an $SO(5,5)$ element as well as their explicit form. Hence we have therefore found an explicit relationship between the symmetries Z_2^0 and Z_2^*.

Now we make connection between some of the results of Ref.[3] and Ref.[4]. For that let us work in metric $\tilde{L}$ of Ref.[4] and spinor multiplet basis, $V_{16}^{-1}\,P_{\bar{\mu}}$. So we will operate on objects associated with spinor representations of Ref.[4] like Z_2^0 and $Z_2^\star$ etc. by V_{16}^{-1}; and use superscript 's' to denote new ones. We will need apply η^{10} and η^{16} matrices on objects associated with vector and spinor representations respectively of Ref.[3]. Now we show that for specific T-duality elements, $\tilde{\Omega}$ relating orientifold pairs of Ref.[4] and $\tilde{\Omega}_0$ relating T-dual pairs of Ref.[3] are equivalent. For that let us consider a T-duality element embedded in $SO(5,5)$ of Ref.[3] in the metric $\tilde{L}$ and spinor multiplet basis, $V_{16}^{-1}\,P_{\bar{\mu}}$. Let us denote its vector and spinor representations as $\tilde{\Omega}^t, \quad \tilde{R_s}^t$. We use superscript 't' to refer T-duality. We see that this element is given in terms of T-duality element of Ref.[3] in the metric $\bar{L}$ as:

$$\tilde{\Omega}^t = \begin{pmatrix} I_2 & 0 \\ 0 & \eta' R_s(\Omega)(\eta')^T \end{pmatrix}, \tilde{R_s}^t = \begin{pmatrix} \eta' R_c(\Omega)(\eta')^T & \\ & \eta'\Omega(\eta')^T \end{pmatrix}. \tag{26}$$

Then applying $\tilde{\Omega}$ and $\tilde{R_s}^s$ on $\tilde{\Omega}^t$ and $\tilde{R_s}^t$ respectively we get the duals (denoted as primed):

$$(\tilde{\Omega}^t)' = \tilde{\Omega}\tilde{\Omega}^t\tilde{\Omega}^{-1},$$

$$
\begin{aligned}
&= \eta^{10}\bar{\Omega}_0 \begin{pmatrix} I_2 & \\ & (R_s(\Omega)^T)^{-1} \end{pmatrix} \bar{\Omega}_0(\eta^{10})^T, && (27)\\
(\tilde{R_s}^{\,t})' &= \tilde{R_s}^{\,s}\tilde{R_s}^{\,t}(\tilde{R_s}^{\,s})^{-1}, \\
&= \eta^{16}R_s(\Omega_0) \begin{pmatrix} (R_c(\Omega)^T)^{-1} & \\ & (\Omega^T)^{-1} \end{pmatrix} R_s(\Omega_0)^{-1}(\eta^{16})^T. && (28)
\end{aligned}
$$

In eqns. (27) and (28) we have used eqns. (2.4), (2.6), (2.15) and (2.30) of Ref.[3]. Now for all the examples of Ref.[3], the matrices $\Omega, R_c(\Omega), R_s(\Omega))$ are orthogonal matrices. Hence for them action of $(\tilde{\Omega}_0, \tilde{R}_s(\Omega_0))$ and $(\tilde{\Omega}, \tilde{R}_s(\Omega))$ are same, or in other words $\tilde{\Omega}$ and $\tilde{\Omega}_0$ are equivalent.

Now let us discuss the constructions of models by simultaneous projections by the T-duality elements of [3] together with Z_2^0 or Z_2^*. Here we note in the context of eq.(26) that the matrices $\Omega, R_c(\Omega)$ and $R_s(\Omega)$ satisfy $\eta^{'}\,(R_s(\Omega), R_c(\Omega), \Omega)\,(\eta^{'})^T = (R_s(\Omega), R_c(\Omega), \Omega)$, when $\Omega, R_c(\Omega), R_s(\Omega)$ are of the form of 8-dimensional matrices $\begin{pmatrix} A & \\ & A \end{pmatrix}$, where A denotes a 4-dimensional diagonal block. Then $\tilde{\Omega}^t$ and $\bar{\Omega}^t$ are identical and so are $\tilde{R}_s^t$ and $\bar{R}_s^t$. For the examples of Ref.[3] the same thing happens with the dual elements also. One such T-duality element in Ref.[3] is $\Omega = (\pi, 0, \pi, 0)$. These pairs of T-duality elements of Ref.[3] are all diagonal. Again $V^{-1}\,\Omega(Z_2^0)\,V$ and $V^{-1}\,\Omega(Z_2^\star)\,V$ are also diagonal. Hence we get unambiguously fields from unprimed multiplets which are invariant under both T-duality element, discussed above and $V^{-1}\,\Omega(Z_2^0)\,V$. In the same way we get from primed multiplets simultaneous invariants under the dual T-duality element and $V^{-1}\,\Omega(Z_2^\star)\,V$ on the other side giving us dual pair of models. By this process of intersection we get models in four, three or two dimensions respectively. One such model in four dimension has been given in [4].

To conclude, we have verified that for construction of typeIIA dual pairs through orientifolding, an explicit U-duality relationship for fields in the vector as well as spinor representation can be constructed. Unlike the pairing of type IIA on K3,with heterotic string on T_4, [16] this relationship has been proven through the construction of appropriate gamma matrices. We can now follow Ref.[12] to test duality of such pair of models. It will be interesting to see how such models fit in the framework of M and F theories [13, 14, 15, 5].

I would like to thank Ashoke Sen for several crucial suggestions and discussions. I am greatly indebted to Alok Kumar for guidence and encouragement throughout the course of this work. I would also like to thank Dileep Jatkar, Rajesh Parwani for discussions, Harvendra Singh, Koushik Roy for critical reading of the manuscript and Mehata Research Institute, Allahabad,for hospitality where this work was started.

References

[1] E. Witten, "String Theory Dynamics in Various Dimensions" *Nucl. Phys.* **B 443** (1995) 85.

[2] C. Vafa, E. Witten, "Dual String Pairs with N=1 and N=2 supersymmetry in Four Dimensions", *Nucl. Phys.* **B** (1996),**hep-th /9507050**.

[3] A Sen, C. Vafa, "Type II Dual Pairs" *Nucl. Phys.* **B 455** (1995) 165-187.

[4] A Kumar, "Orientifold and Type II Dual Pairs", **hep-th /9601067**(to appear in *Phys. Rev.* **D**).

[5] A Sen, "F-theory and Orientifolds", **hep-th /9605150**.

[6] R. Dijkgraph, E. Verlinde, H.Verlinde, "BPS spectrum of the five brane and black hole entropy", **hep-th /9603126**, " BPS quantization of the five brane", **hep-th /9604055**.

[7] J. J. Sakurai, "Advanced Quantum Mechanics" P-97; C. Itzykson, J. B. Zuber, "Quantum Field Theory" P-51.

[8] J. F. Cornwell, "Group Theory in Physics, Vol-I appendix and Vol-II".

[9] M. B. Green, J. H. Schwarz, E. Witten, "String Theory Vol-I ,Chapter-V".

[10] C. Wetterich , "Massless spinors in more than four dimensions", *Nucl. Phys.* **B 211** (1983) 177-188.

[11] J. H. Schwarz, "Supergravity in various dimensions and the implication", *Phys. Lett.* **B 95** (1980) 219.

[12] A Sen, "U-duality and intersecting P-branes" *Phys. Rev.* **D 533** (1996) 2874-2894.

[13] J. H. Schwarz, "The Power of M Theory", **hep-th /9510086**.

[14] C. Vafa, "Evidence for F-theory", *Nucl. Phys.* **B 469** (1996) 403-418.

[15] D. Morrison, C. Vafa, "F-theory on Calabi-Yau Threefoldsds", **hep-th /9602114**, **hep-th /9603161**.

[16] H. Gao, "More Dual Pairs from Orbifolding", **hep-th /9512060**.